

# **A Review on Image Segmentation by Fuzzy C-Means Clustering Algorithm**

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**ABSTRACT:** In this work we present a detailed study and comparison of some Fuzzy C-means Clustering based on segmentation algorithms. Fuzzy C-Means (FCM) algorithm, Enhanced FCM (EnFCM), spatially weighting FCM(SWFCM) have been used for comparison. These three algorithms are studied and analyzed by implementing on Synthetic Aperture Radar (SAR) images with Gaussian and salt and pepper noise for comparison and review. The time taken by segmentation of an image is dependent on the image size, and hence the large size image takes more segmentation time.

**KEYWORDS:** Segmentation, Fuzzy C-Means (FCM), Enhanced FCM (EnFCM), Spatially Weighting FCM(SWFCM)

## **I. INTRODUCTION**

Image segmentation has an important role in image analysis. The goal of image segmentation is partitioning the image into a set of disjoint regions with uniform and homogeneous attributes such as intensity, colour, texture etc. The image segmentation can be divided into thresholding, region extraction, edge detection and clustering. In image processing, two terms are usually seen very close to each other: clustering and segmentation. When we analyse the colour information of the image, e.g. trying to separate regions or ranges of colour components having same characteristics, the process is called colour clustering and mapping the clusters onto the spatial domain by physically separated regions in the image is called segmentation. In images, the boundaries between objects are blurred and distorted due to the imaging acquisition process. Furthermore, object definitions are not always crisp and knowledge about the objects in a scene may be vague. Fuzzy set theory and Fuzzy logic are ideally suited to deal with such uncertainties. Fuzzy sets were introduced in 1265 by Lofti Zadeh[1] with a view to reconcile mathematical modeling and human knowledge in the engineering sciences. The Fuzzy C-means (FCM) [2], algorithm, proposed by Bezdek, is the most widely used algorithm in image segmentation. FCM has been successfully applied to feature analysis, clustering, and classifier designs in fields such as astronomy, geology, medical imaging, target recognition, and image segmentation. An image can be represented in various feature spaces and the FCM algorithm classifies the image by grouping similar data points in the feature space into clusters. In case the image is noisy or distorted, FCM technique wrongly classify noisy pixels because of its abnormal feature data which is the major limitation of FCM. Various approaches are proposed by researchers to compensate this drawback of FCM.

Tolias and Panas developed a rule based system by imposing spatial continuity constraints and using the effect of an eight-connected neighbourhood to smooth the noise [3]. Acton and Mukherjee[4] use multiscale information to enforce spatial constraints. Dave[5] proposed the idea of a noise cluster to deal with noisy data using Noise Clustering. He separated the noise into one cluster but this technique is not suitable for image segmentation, as we cannot separate pixels from the image. Another similar technique, PCM, proposed by Krishnapuran and Keller [6] interpreted clustering as a possibilistic partition. However, it caused clustering being stuck in one or two clusters. Ahmed et al.[7] modified the objective function of FCM by introducing a term that allowed the labelling of a pixel to be influenced by the labels in its immediate neighbourhood. Zhang Yang, Fulai Chuang et al. (2002)[8] developed a robust clustering technique by deriving a novel objective function for FCM. Jiayin Kang et al.[9] proposed

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another technique which modified FCM objective function by incorporating the spatial neighbourhood information into the standard FCM algorithm. Y. Yang et al.[10] proposed a novel penalized FCM (PFCM) for image segmentation where penalty term acts as a regulators in the algorithm which is inspired by neighbourhood maximization (NEM) algorithm[11]. On the other hand, there is a trend in recent machine use kernel method for constructing a nonlinear version of linear algorithms. e.g. SVM [12—14], KPCA [15] and KFD[16]. And this kernel method has also been applied to unsupervised clustering [17—21] like FCM and a novel kernelized fuzzy C-means (KFCM) algorithm is proposed.

## II. METHODOLOGY

### Standard FCM

Fuzzy c-means (FCM)[1] is a method of clustering which allows one piece of data to belong to two or more clusters. This method developed by Dunn and improved by Bezdek is frequently used in pattern recognition. FCM is the most popular fuzzy clustering algorithm. It assumes that number of clusters 'c' is known in priori and minimizes the objective function (JFCM) as:

$$Obj_{fcM} = \min_{(U, V)} \left\{ J_m(\tilde{U}, V) = \sum_{i=1}^c \sum_{k=1}^n \mu_{ik}^{m'} (d_{ik})^2 \right\} \quad (1)$$

Where  $d_{ik} = \|x_k - v_i\| = \left[ \sum_{j=1}^m (x_{kj} - v_{ij})^2 \right]^{\frac{1}{2}} \quad (2)$

And where  $\mu_{ik}$  is membership of the  $k$ th data point in the  $i$ th class,  $m'$  is a weighing parameter. This value has a range  $m' \in [1, \infty)$ . Here  $n$  is the number of data set. And  $v_i$  is the  $i$ th cluster center. Which is described by  $m$  feature ( $m$  coordinates) and can be arranged in vector form as  $v_i = \{v_{i1}, v_{i2}, \dots, v_{im}\}$ . Cluster center is calculated as:

$$v_i = \frac{\sum_{k=1}^n \mu_{ik}^m x_{ki}}{\sum_{k=1}^n \mu_{ik}^m} \quad (3)$$

Where  $j$  is a variable on the feature space, that is,  $j = 1, 2, \dots, m$ .

And update the partition matrix for the  $r$ th step as:

$$\mu_{ik}^{(r+1)} = \left[ \sum_{j=1}^c \left( \frac{d_{ik}^{(r)}}{d_{jk}^{(r)}} \right)^{2/(m'-1)} \right]^{-1} \quad \text{for } I_k \neq \emptyset \quad (4)$$

or  $\mu_{ik}^{(r+1)} = 0$ , for all classes  $i$  where  $i \in \tilde{I}_k$ . (5)

where  $I_k = \{i | 2 \leq c < n; d_{ik}^{(r)} = 0\}$ ,  $\tilde{I}_k = \{1, 2, \dots, c\} - I_k$ ,

and  $\sum_{i \in I_k} \mu_{ik}^{(r+1)} = 1$ . (6)

FCM algorithm proposed by Bezdek was as follow:

1. Fix  $c$  ( $2 \leq c < n$ ) and select a value for parameter  $m'$ . Initialize the partition matrix  $\tilde{U}^{(0)}$ . Each step in this algorithm will be labelled  $r$ , where  $r = 0, 1, 2, \dots$ . Set  $\epsilon_L$  for convergence.
2. Calculate the  $c$  centers  $\{v_i^{(r)}\}$  by Eq. (3) for each step.
3. Update the partition matrix for the  $r$ th step,  $\tilde{U}^{(r)}$  by Eq.(4) and (5).
4. If  $\|\tilde{U}^{(r+1)} - \tilde{U}^{(r)}\| \leq \epsilon_L$ , stop; otherwise set  $r = r+1$  and return to step 2.

In step 4, we find a matrix norm  $\|\cdot\|$ . This algorithm iterates up to stopping condition to determine whether the solution is good enough. In step 3 Eq.(5) is applicable when the variable  $d_{jk}$  is zero. Since this variable is in the denominator of a fraction, the operation is undefined mathematically, and computer calculations are abruptly halted. If a zero value is detected, Eq. (5) sets the membership for that partition value to be zero. Equation (6) simply says that all the nonzero partition elements in each column of the fuzzy classification partition,  $\tilde{U}$  sum to unity.

### Spatial FCM algorithms

The conventional FCM algorithm works well on noise-free images but very sensitive to noise but noise occurs very often in real images. This sensitiveness is due to absence of consideration of the spatial context of each pixel.

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### FCM\_S algorithm

In [7], Ahmed et al. modify the original objective function by adding a penalty term that allows the memberships of each pixel  $x_k$  to be influenced by its neighbourhood. The new objective function is defined as

$$J_m = \sum_{i=1}^c \sum_{k=1}^n \mu_{ik}^{m'} \|x_k - v_i\|^2 + \frac{\alpha}{N_R} \sum_{i=1}^c \sum_{k=1}^n \mu_{ik}^{m'} \sum_{r \in N_k} \|x_r - v_i\|^2 \quad (7)$$

$N_k$  = the set of neighbours in a window around the pixel  $x_k$ .  $N_R$  is the cardinality of  $N_k$ , and  $\alpha$  is the balance parameter, allows to control the effect of the neighbouring terms. In this algorithm, the computing the neighbouring terms in each iteration is computationally expensive and tuning of  $\alpha$  is not easy.

Two necessary but not sufficient conditions for  $J_m$  to be at its local extreme will be obtained as follows:

$$\mu_{ik} = \frac{\left( \|x_k - v_i\|^2 + \frac{\alpha}{N_R} \sum_{r \in N_k} \|x_r - v_i\|^2 \right)^{-1/(m-1)}}{\sum_{i=1}^c \left( \|x_k - v_i\|^2 + \frac{\alpha}{N_R} \sum_{r \in N_k} \|x_r - v_i\|^2 \right)^{-1/(m-1)}} \quad (8)$$

$$v_i = \frac{\sum_{k=1}^n \mu_{ik}^{m'} (x_{ki} + \frac{\alpha}{N_R} \sum_{r \in N_k} x_r)}{(1+\alpha) \sum_{k=1}^n \mu_{ik}^{m'}} \quad (9)$$

This algorithm is called FCM\_S. Although steps of this algorithm is same as FCM only Eq. are changed. The second term in the numerator of (9) is in fact a neighbour average gray value around  $x_r$ , the image composed of all the neighbour average values around all the image pixels forms a so-called local neighbour average image or equivalently mean-filtered image. A shortcoming of FCM\_S is that computing the neighbour term will take much time in each iteration step. However during each iteration, all the pixels of the image are considered, leading to a large computation time. Chen and Zhang [22] proposed a variant of FCM\_S, FCM\_S1, which simplified the neighbourhood term of FCM\_S. And the low-complexity objective functions.

### FCM\_S1 algorithms

In [5], the authors propose to reduce the computation time of the solutions derived from (7) by computing in advance the mean value of the pixel within the specified window. The Objective function is:

$$J_s = J_m + \sum_{i=1}^c \sum_{k=1}^n \mu_{ik} \|x_k - v_i\|^2 + \alpha \sum_{i=1}^c \sum_{k=1}^n \mu_{ik}^m \|\bar{x}_i - v_k\|^2 \quad (10)$$

$$\mu_{ik} = \frac{\left( \|x_k - v_i\|^2 + \frac{\alpha}{N_R} \sum_{r \in N_k} \|\bar{x}_k - v_i\|^2 \right)^{-1/(m-1)}}{\sum_{i=1}^c \left( \|x_k - v_i\|^2 + \frac{\alpha}{N_R} \sum_{r \in N_k} \|\bar{x}_k - v_i\|^2 \right)^{-1/(m-1)}} \quad (11)$$

$$v_i = \frac{\sum_{k=1}^n \mu_{ik}^{m'} (x_{ki} + \alpha(\bar{x}_k))}{(1+\alpha) \sum_{k=1}^n \mu_{ik}^{m'}} \quad (12)$$

where  $\bar{x}_k$  is the mean of neighboring pixels in the window around  $x_k$ . Additionally, they propose another algorithm FCM\_S2 its objective function is  $J_{s2}$  where  $\bar{x}_i$  is the median value of the neighboring pixels.

FCM\_S, FCM\_S1 and FCM\_S2 algorithms steps are as follow:

1. Fix  $c$  ( $2 \leq c < n$ ) and select a value for parameter  $m'$ . Initialize the partition matrix  $\widetilde{U}^{(0)}$ . Each step in this algorithm will be labelled  $r$ , where  $r=0, 1, 2, \dots$ . Set  $\varepsilon_L$  for convergence.
2. For FCM\_S1 and FCM\_S2 only, compute the mean or median filtered image.
3. Update the partition matrix for the  $r$  th step,  $\widetilde{U}^{(r)}$  by Eq. (8) for FCM\_S or Eq.(11) for FCM\_S1 and FCM\_S2
4. Calculate the  $c$  centers  $\{v_i^{(r)}\}$  by Eq.(9) for FCM\_S or FCM\_S1 and FCM\_S2 as Eq. (12) for each step.
5. If  $\|\widetilde{U}^{(r+1)} - \widetilde{U}^{(r)}\| \leq \varepsilon_L$ , stop; otherwise set  $r = r+1$  and return to step 2.

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Then, they introduce the use of kernel-induced distances instead of the usual Euclidean one. The corresponding algorithms are respectively denoted as KFCM\_S1 and KFCM\_S2 in the sequel. Moreover, since they use kernel-induced distance, they also propose to automatically set the parameters of the Gaussian kernel. The corresponding algorithm is claimed to be a generalized version of KFCM. Here again, the authors allow to consider mean-based or median-based spatial filtering. The derived algorithms are respectively denoted as GKFCM\_S1 and GKFCM\_S2 in the sequel. But all are suffers of the same high computation time .

### Speeded up clustering algorithms

The main drawback of such iterative clustering algorithms is their running time. In [23], the authors propose a fast and accurate clustering method of images. The time reduction is operated by aggregating similar examples and using the weighted prototype in the clustering, giving the brFCM algorithm. In order to speed up the segmentation, Szilagy et al.[24] used the idea of Eschrich et al. [23] to propose the EnFCM algorithm

### EnFCM algorithm

In order to speed up the segmentation, L.Szilágyi et al. [24] proposed the EnFCM algorithm to speed up the segmentation process for gray level image. In order to accelerate FCM\_S, a linearly-weighted sum image  $\xi$  is in advance formed from the original image and its local neighbour average image . They first construct a linearly-weighted sum image with local neighbours of each pixel as follows:

$$\xi_k = \frac{1}{(1 + \alpha)} \left( x_k + \frac{\alpha}{N_k} \sum_{j \in N_k} x_j \right) \quad (13)$$

where  $\xi_k$  denote the gray value of the  $k$ th pixel of the image  $\xi$ ,  $x_j$  represents the neighbours of  $x_k$ ,  $N_k$  stands for the set of neighbours falling into a window around  $x_k$  and  $\sum_{j \in N_k} x_j$  plays again the same role as mean-filtered pixel value .The  $\alpha$  plays the same role as before. Then the fast segmentation method is performed on the gray level histogram of the generated image  $\xi$  concretely, the objective function used for fast segmenting the newly-generated image  $\xi$  is defined as:

$$J_m = \sum_{i=1}^c \sum_{l=1}^q \gamma_l \mu_{il}^{m'} \|\xi_l - v_i\|^2 \quad (14)$$

Where  $\xi = \{\xi_l, l=1,2, q\}$  is the data set rearranging from the image  $\xi$  defined in Eq.(13) according to gray level.  $\{v_i\} (i=1,2,..q)$  represents the prototype of the  $i$ th cluster.  $\mu_{il}^{m'}$  represents the fuzzy membership of gray value  $l$  with respect to cluster  $i$  .  $q$  is the number of the gray level of the given image which is generally much smaller than  $N$ .  $\gamma_l$  is the number of the pixels having the gray value equal to  $l$ , where  $l=1, 2, , q$ . and

$$\sum_{l=1}^q \gamma_l = N.$$

And under the constraints that

$$\sum_{i=1}^c \mu_{il} = 1$$

for any  $l$ , minimize  $J_m$ . Specifically, taking the first derivatives of  $J_m$  with respect to  $\mu_{il}$  and  $v_i$  and find their equation respectively, two necessary but not sufficient conditions for  $J_s$  to be at its local extrema will be obtained as follows :

$$\mu_{il} = \frac{(\xi_l - v_i)^{-2/(m-1)}}{\sum_{r=1}^c (\xi_l - v_r)^{-2/(m-1)}} \quad (15)$$

$$v_i = \frac{\sum_{l=1}^q \gamma_l \mu_{kl}^{m'} \xi_l}{\sum_{k=1}^n \gamma_l \mu_{kl}^{m'}} \quad (16)$$

### EnFCM Algorithm

1. Fix  $c$  ( $2 \leq c < n$ ) and select a value for parameter  $m'$ . Initialize the partition matrix  $\tilde{U}^{(0)}$ . Each step in this algorithm will be labelled  $r$ , where  $r=0, 1, 2, \dots$  and then select initial class prototypes, set  $\epsilon > 0$  to a very small value.
2. Compute the new image  $\xi$  in terms of (13) in advance.
3. Update the partition matrix using (15).
4. Update the prototypes center using (16).

Repeat Steps3–4 until the following termination criterion is satisfied:

$$\|V_{new} - V_{old}\| < \epsilon ,$$

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where  $V=[v_1, v_2, \dots, v_c]$  are the vectors of cluster prototypes.

EnFCM provides the significant reduction of execution time due to instead of using total data attributes ,use the gray levels of given image. Which is usually overlooked in many FCM-type algorithms including FCM, FCM\_S, FCM\_S1 and FCM\_S2 . Due to that the gray value of the pixels is generally encoded with 8 bit resolution, equivalently, 256 levels in total, and thus the number  $q$  of gray levels is generally much smaller than the size  $N$  of the image. Consequently, the time complexity is drastically reduced especially for the large image.

### III. RESULTS AND COMPARISON

In these experiments, first execute the segmentation algorithms FCM, FCM\_S1 and EnFCM on an original SAR image Then perform the three segmentation algorithms on noisy SAR images corrupted, respectively, by Gaussian noise and salt and pepper noise to investigate the robustness of the algorithms. Finally, the corresponding quantitative comparisons are presented. In all the following experiments, we set the parameters  $c=2, c=5, m=2, a=5, \epsilon = 0.001$ . (CPU: 1.66GHz, Memory: 2GB, Operating system: windows vista, Soft ware: Matlab 13.0).

#### Results on Original SAR Image

To compare the segmenting performances of the above three algorithms, FCM, FCM\_S1 and EnFCM, we applied these algorithms to an original test SAR image as shown in Figure 1(a) with the size of pixels(400\*321), and the corresponding segmenting results are, respectively, displayed in Figures 1(b,c,d,e,f,g) using  $c=2$  and  $c=5$  .

In figure:1 FCM, FCM\_S1 and EnFCM algorithm gives more over same visually significant result . More detailed quantified comparison according to execution time and iteration step are given in the next subsection where we can detect difference between them.

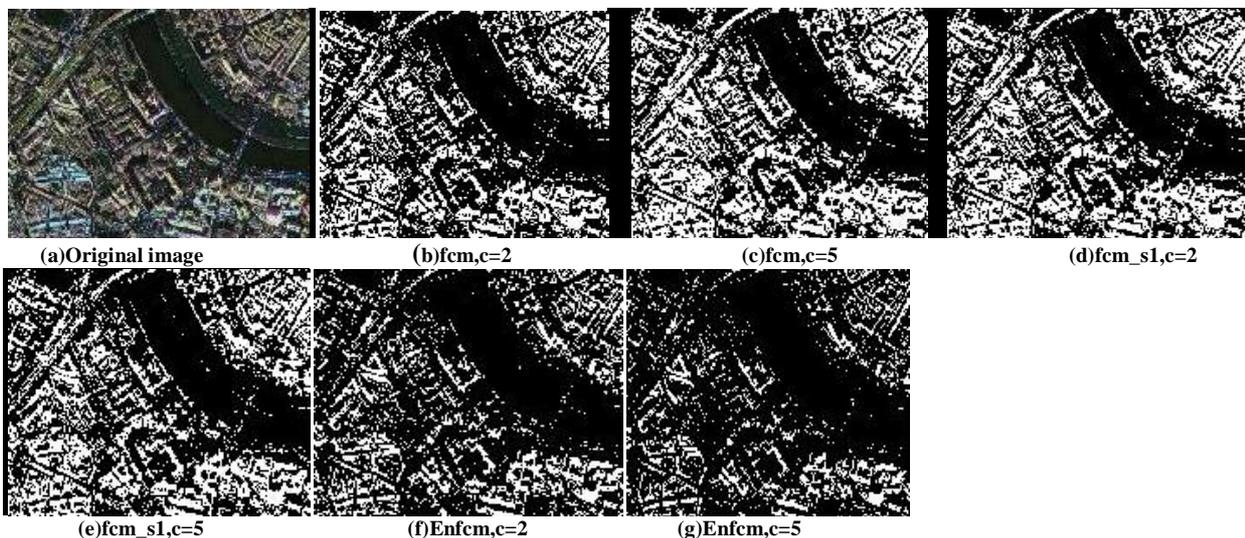


Figure 1. Segmentation results on SAR image without noise .(a) is original (b)-(g) is segmented image:

#### Results on SAR Images Corrupted by Noises

To examine the above three algorithms 'robustness to noise' I apply the three algorithms on SAR images corrupted by noises. Figure 2(a) is the original image corrupted by Gaussian noise ( $\mu = 0, \sigma = 0.05$ ) and The segmented results on Figures 2(a) are shown in Figures 2(b)to2(g).Using above mention algorithms and taking  $c=2$  and  $c=5$  for better analysis. Here, we can detect difference between them.FCM and FCM\_S1 is influenced by the Gaussian noise But EnFCM give good result, less affected .If we increase number of center they give more visually significant result.

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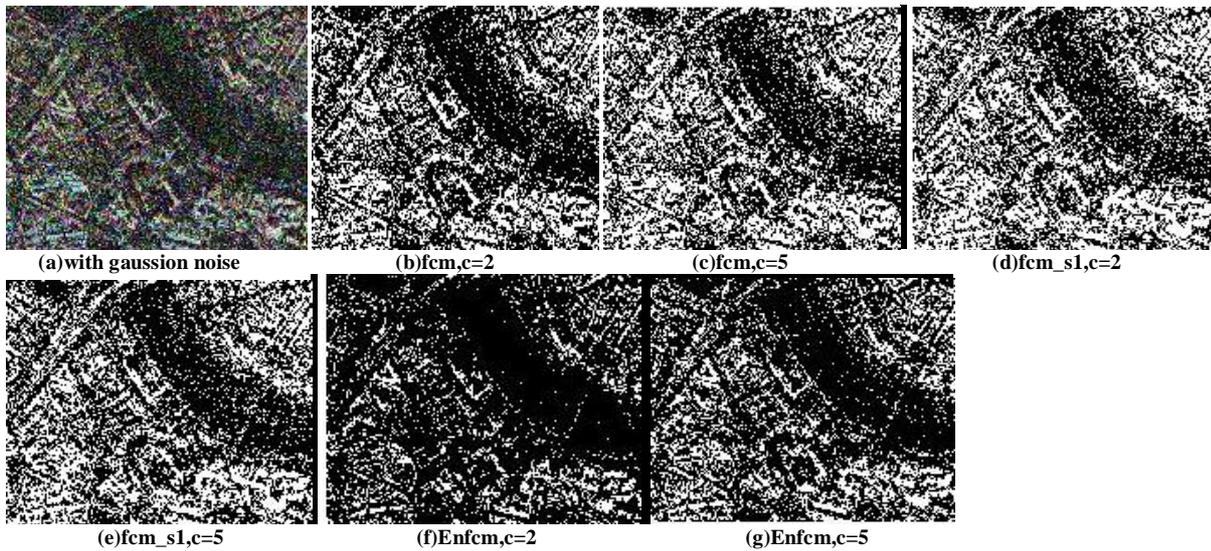


Figure 2. Segmentation results on SAR image corrupted by Gaussian noise .(a) is original (b)-(g) is segmented image:

Figure 3(a) is the original image corrupted by salt and pepper noise ( $d = 0.05$ ) and The segmented results on Figures 3(a) are shown in Figures 2(b)to2(g). Here, FCM\_S1 give better result than others .And EnFCM also give clearly visible segmentation.

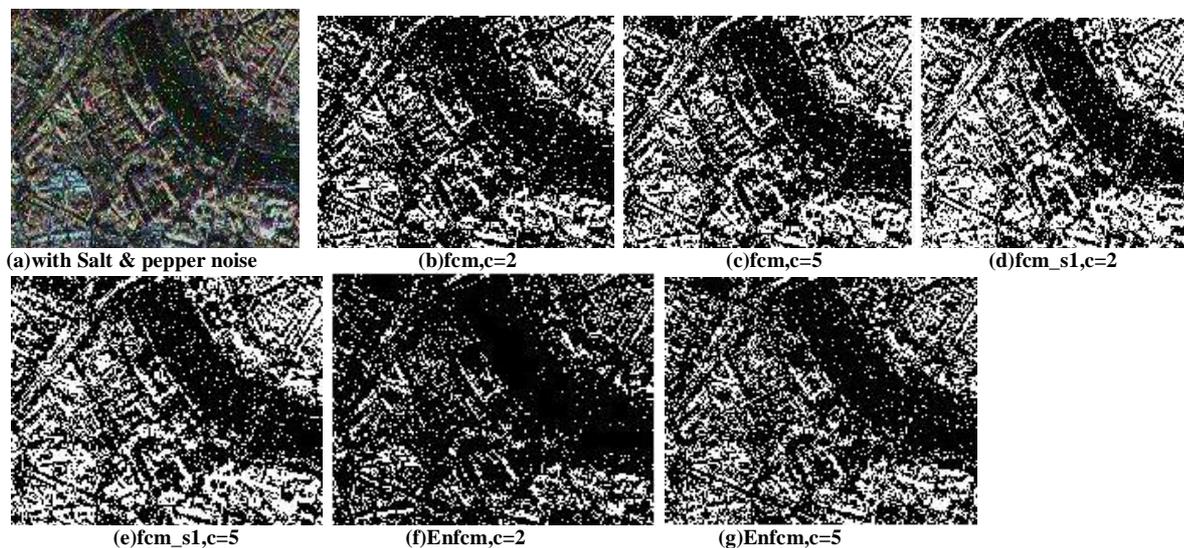


Figure 3. Segmentation results on SAR image corrupted by salt and pepper noise.(a) is original (b)-(g) is segmented image:

### Tabulate quantitative segmenting result

From Tables 1 we can see that the execution time of EnFCM is far less than that of FCM and FCM\_S1. Furthermore, the iteration count of EnFCM is equal or less compared with FCM and FCM\_S1. Due to dependence on the number of the gray-level both the iterations and execution time will be less. Because the number of gray-levels( $q$ ) are always less than the total pixels( $400 \times 321$ ) in SAR image.

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**Table 1: Execution time and Iteration count of three algorithm without noise using two and five centres.**

Algorithms	C=2		C=5	
	Ex. Time	Itr. count	Ex. Time	Itr. count
FCM	8.81	24	77.85	89
FCM_S1	5.92	25	47.96	85
EnFCM	0.23	22	0.32	85

Now from table 2 and 3 ,after corrupting by Gaussian noise and salt and pepper, Iteration count is more than others but EnFCM have least execution time.

**Table 2: Execution time and Iteration count of three algorithm with Gaussian noise using two and five centres**

Algorithms	C=2		C=5	
	Ex. Time	Itr. count	Ex. Time	Itr. count
FCM	9.52	26	80.93	93
FCM_S1	6.76	30	33.03	88
EnFCM	0.20	35	0.33	99

**Table 3: Execution time and Iteration count of three algorithm with salt and pepper noise using two and five centres**

Algorithms	C=2		C=5	
	Ex. Time	Itr. count	Ex. Time	Itr. count
FCM	9.42	26	84.91	92
FCM_S1	6.14	26	46.08	80
EnFCM	0.18	35	0.32	88

## IV.CONCLUSIONS

In this work we discussed the performance of the three algorithms FCM, FCM\_S1, EnFCM for the synthetic aperture radar images in noiseless case and when corrupted with salt and pepper and Gaussian noise. We compare these algorithms based on execution time and iteration count. It has been observed that execution time and iteration count for EnFCM is better than FCM and FCM\_S1 under noiseless condition. It also observed that FCM algorithm lacks enough robustness under the noisy condition like Gaussian noise and the Salt and pepper noise, while EnFCM and FCM\_S1 can basically eliminate the effect of the noises. Here also EnFCM have least execution time. Also as expected execution time is lesser with two clusters than with five clusters but in case of five clusters difference is clearly visible. Only execution time and iteration count is not sufficient for performance evolution So our Future work is improving the accuracy, precision, and computation speed of the segmentation algorithms, using numerous metrics like Probabilistic Rand Index (PRI), Variation of Information (VOI), The Global Consistency Error (GCE), and The Boundary Displacement Error (BDE)

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